Title: Leap Into Exponential Functions

Brief Overview:

Students will be able to transition from quadratic functions to exponential. After noting major differences between the two, they will develop the concept of the multiplier using the TI-83 Plus regression feature and the TI-Interact application. This activity will ultimately lead to the standard equation of an exponential function of both growth and decay. Given a set of data, the students will be able to analyze it and draw conclusions about it.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

<u>Number and Operations</u>

- Students will understand numbers, ways of representing numbers, relationships among numbers, and number systems

<u>Algebra</u>

- Students will understand patterns, relations, and functions;
- Students will represent and analyze mathematical situations and structures using algebraic symbols;
- Students will use mathematical models to represent and understand quantitative relationships;
- Students will analyze change in various contexts

Data Analysis and Probability

- Students will formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- Students will select and use appropriate statistical methods to analyze data;
- Students will develop and evaluate inferences and predictions that are based on data

- **Process Standards**

Problem Solving

- Students will build new mathematical knowledge through problem solving;
- Students will monitor and reflect on the process of mathematical problem solving

Communication

- Students will organize and consolidate their mathematical thinking through communication;
- Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- Students will use the language of mathematics to express mathematical ideas precisely

Connections

- Students will recognize and use connections among mathematical ideas;
- Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole

Representation

- Students will create and use representations to organize, record, and communicate mathematical ideas;
- Students will select, apply, and translate among mathematical representations to solve problems;
- Students will use representations to model and interpret physical, social, and mathematical phenomena

Links to Maryland High School Mathematics Core Learning Units:

Functions and Algebra

- The student will recognize, describe, and/or extend patterns and functional relationships that are expressed numerically, algebraically, and/or geometrically.
- The student will represent patterns and/or functional relationships in a table, as a graph, and/or by mathematical expression.
- The student will describe the graph of a non-linear function and discuss its appearance in terms of the basic concepts of maxima and minima, zeros (roots), rate of change, domain and range, and continuity.
- The student will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

Data Analysis and Probability

- The student will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.
- The student will make informed decisions and predictions based upon the results of simulations and data from research.

Grade/Level:

Grades 10-12; Algebra 2

Duration/Length:

Three 90 minute class periods. (Adaptable to 50 minute classes.)

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- General characteristics of quadratic functions
- Basic knowledge of TI-83 graphing calculator
- Ability to draw and support conclusions from a data set

Student Outcomes:

Students will be able to:

- Determine whether a data set is best represented by a quadratic or exponential function.
- Distinguish between an exponential and non-exponential expression.
- Determine the multiplier and then the standard form of exponential functions.
- Explain why an expression is exponential growth or decay.
- Given a set of data, perform the appropriate regression analysis, and support conclusions.

Materials/Resources/Printed Materials:

- TI-83 Plus calculator with TI-Interact application and overhead projector
- Student worksheets 1, 2, 3 and 4, Group Data Worksheets 1-4, Student Assessment
- Technology Worksheet
- Poster paper
- Internet Access for a computer that can project the screen for students to see

Development/Procedures:

After students begin with a brainstorming activity to recall characteristics of quadratic functions/graphs, they will perform a regression on a set of data which is quadratic to find the curve of best fit. Then they will create a scatterplot for a set of data which is

exponential. For the exponential data, they will answer questions and apply a quadratic regression analysis. By making a comparison of the graphs and data tables from both sets of data, the students will conclude that it is not quadratic and, consequently, be asked to look for different features in the second set of data. By completing a table for the differences in output, students will discover the multiplier in exponential data. Discussion will lead to the standard form of the equation. Using TI-Interact, students will assess the accuracy of their equation and then move on to evaluate the effect of changing *a* and *b* values on the graph. A discussion about the exponential decay function will follow the exploration with TI-Interact. For classwork students will perform a regression analysis on two sets of data to determine the function and curve of best fit. Homework will be a worksheet with problems on the basic concepts.

The second day will begin with a discussion of the homework. Then the class will divide into four groups. Each group will use sample data to perform a regression analysis, produce a poster of the results, and present the findings. Students will be assigned an exit ticket asking them to write down any questions they still have about exponential functions.

On day three the questions from the exit tickets will be addressed. Then the students will take the assessment.

Assessment:

Assessment will be done in various forms throughout the lesson. Informal assessments will be the classwork and homework worksheets from Day 1 and the group posters from Day 2. A formal assessment will be given at the end of the unit, which will include several short answer questions and one extended constructed response. The rubric to be used to grade the ECR will be the same as the Maryland High School Assessment ECR Rubric.

Extension/Follow Up:

You can use the results from the second day group activity to lead into a discussion on *e* since some of the data was generated using formulas with *e*. The data sets that have *e* as the multiplier are #1 and #4.

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Teacher Notes: Leap into Exponential Functions

Day 1 – Quadratics vs. Exponentials

Objective: The student will be able to

- determine whether a data set is best represented by a quadratic or exponential function
- distinguish between an exponential and non-exponential expression.
- determine the multiplier and then the standard form of exponential functions
- explain why an expression is exponential growth or decay.
- given a set of data, perform the appropriate regression analysis, and support conclusions.

(Designed for a 90 minute class period)

Materials:

- TI-83 Plus calculator with TI-Interact installed
- Student Worksheets #1, 2, 3, and 4

Homework: Student Worksheet #4.

Procedures:

Warm-up: List all that you remember about quadratic functions. (shape of graph, pattern in data, etc.)

Engagement: Have students contribute their ideas by writing them on the board. Continue until all major ideas are included. Discuss the important features like the shape of the graph, standard form of the equation, effect of *a*, pattern in output, and applications.

Exploration: Pass out Student Worksheet #1 for students to complete along with the teacher. Pass out the Technology Worksheet at this time for the students to reference. They will be creating a scatterplot and performing a regression with the data given. Circulate around the room to help students through the process.

Extension: Now pass out Student Worksheet #2 for students to complete. They should be able to work on this in pairs at this point up to #9 with little assistance. Monitor progress; when class has completed up through #9 bring them together for completing the remainder of the worksheet as a class.

Explanation: Discuss the answer to #9 and give them an accurate definition, then continue with steps on this sheet as a whole class. Demonstrate TI-Interact using the overhead calculator.

Evaluation: Give the class Student Worksheet #3 to complete entirely on their own. Towards the end of the period, pass out Student Worksheet #4, which is the homework.

Teacher Notes: Leap into Exponential Functions

Day 2 – Exponential Regression

Objective: The student will be able to

- given a set of data, perform the appropriate regression analysis, and support conclusions in groups.
- communicate the results through a graph and table on a poster

(Designed for a 90 minute class period)

Materials:

- TI-83 Plus calculator
- 4 sets of exponential data, store in page in protectors for use with each class
- Poster paper or large flip chart and markers

Homework: Review for quiz

Procedures:

Warm-up: Did you encounter any problems on the homework? If so what are they?

Engagement: Discuss questions about homework and go over correct answers. Before group work cue up the website below to reiterate the general idea of exponential growth and the multiplier.

(www.jump.net/~otherwise/population/exponent.html - go to the Run applet button on the page to run the demo.)

Exploration: Divide the class into 4 groups. Hand each group their own set of data. Explain to the students that they will be performing a regression analysis just as they did in the previous class. Give directions about the poster they are to produce. It should have a graph showing the scatterplot of the data as well as the curve of best fit with appropriate labels and scales, the equation for the curve of best fit, and the table of data. They will determine the multiplier and include that on their posters along with a justification as to their conclusion about the curve of best fit.

Explanation: Students will present their findings to the class. If time runs out, remaining groups could present in next class or just not present. Therefore, it would be a good idea to make sure groups with data that produces *e* as the multiplier should go first to be sure that observation is available for an extension made in the future.

Evaluation: Have students complete an Exit Ticket which asks them to list any questions they still have about exponential functions.

Note: There is a fifth data set for use as extra credit for any groups that get done early.

Teacher Notes: Leap into Exponential Functions

Day 3 – Assessment time

Objective: The student will be able to

- demonstrate an understanding of exponential functions and regression analysis by completing the assessment.

(Designed for a 90 minute class period)

Materials:

- TI-83 Plus calculator
- Assessment

Procedures:

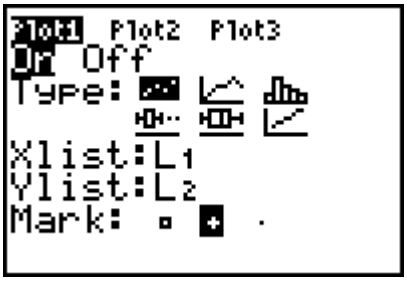
Engagement: Review questions from exit tickets and their answers.

Evaluation: Be sure that each student has a calculator and then pass out the assessment and a copy of the scoring rubric. Students may use the Technology Sheet.

Technology Sheet

Graphing a scatter plot:

1) Press [Y =] and make sure that there are no equations in there and no plots are highlighted. (To turn off any plots, simply highlight and hit [ENTER] to turn off.)
2) To enter the data:
   a) Press [STAT] then go to 1:Edit and press [ENTER]
   b) Clear out L1 and L2 by highlighting the list and pressing [CLEAR] and then the down arrow.
   c) In L1, enter the first set of data
   d) In L2, enter the second set of data
   e) Be sure to check the accuracy of your data, then press [2nd] [MODE]
3) To graph the data:
   a) Press [2nd] [Y =]
   b) Choose 1:Plot1
   c) Fix your screen to look like this→
   d) Press [ZOOM] and choose 9:Z Stat



Regression: curve of best fit:

1) Press [STAT] go over to CALC and choose 5:QuadReg for Quadratic or 0:ExpReg for Exponential
2) Press [2nd] [1] , [2nd] [2] , [VARS] choose [Y - VARS] 1:Function [Y1] [ENTER]
3) Press [GRAPH] to see data with line of best fit and [Y =] to get the equation.

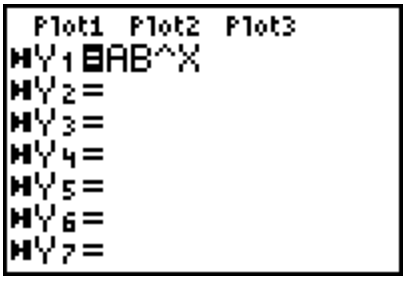
Starting and Ending the Interact Application on the TI-83 Plus:

1) To start the application
   a. Press the [APPS] button. Choose Interact.
   b. A screen will pop up. Press any key to continue. (If you get a message with a choice to 1:Uninstall or 2: Continue, choose to continue.)
   c. You are now in Interact mode.
2) To end application
   a. Press the [APPS] button. Choose Interact.
   b. It will ask you to either 1:Uninstall or 2: Continue, choose to Uninstall.
   c. You are now out of Interact mode.

Technology Sheet

Using Interact to demonstrate exponential functions:

1) Press [Y =] and make sure that there are no equations in there and no plots are highlighted. (To turn off any plots, simply highlight and hit [ENTER] to turn off.)
2) Type in the equation AB^X using the ALPHA keys.
3) Hit [GRAPH] to go to your screen with the data already on it .
4) You may not see your graph until you change the values of A and B.



5) To change the values of A and B, simply highlight the appropriate variable and type in the value you would like and hit [ENTER]. Or you can left and right arrow to increase or decrease your current value.

Student Worksheet #1 – Quadratic Regression

Name: _____ Date: _____

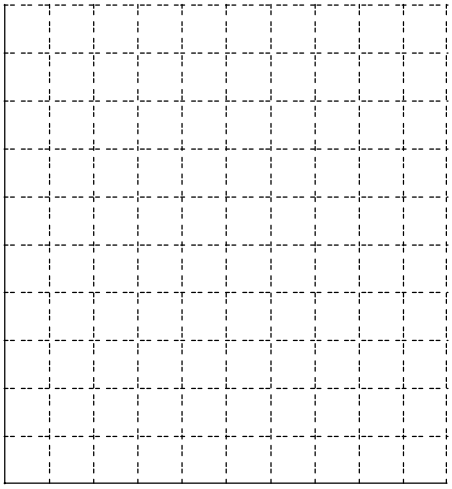
Experiment:

The following data were collected by measuring the stopping distances at varying speeds on dry concrete.

Speed (mph)	Stopping Distance (feet)
20	44
30	83
40	132
50	193
60	264
70	347

Refer to the Technology Worksheet to perform all calculator features.

1) Create a scatterplot of the data above on the TI-83. (Let the stopping distance be our dependant variable.)
2) Sketch a graph of the scatterplot below. Be sure to include labels and scales.



3) What function do you think best fits the graph of these data?

4) Perform a quadratic regression on the data using your TI-83. Sketch the regression line on your graph above and state your regression equation below.

_____

Student Worksheet #2 – Regression Analysis

Name: _____ Date: _____

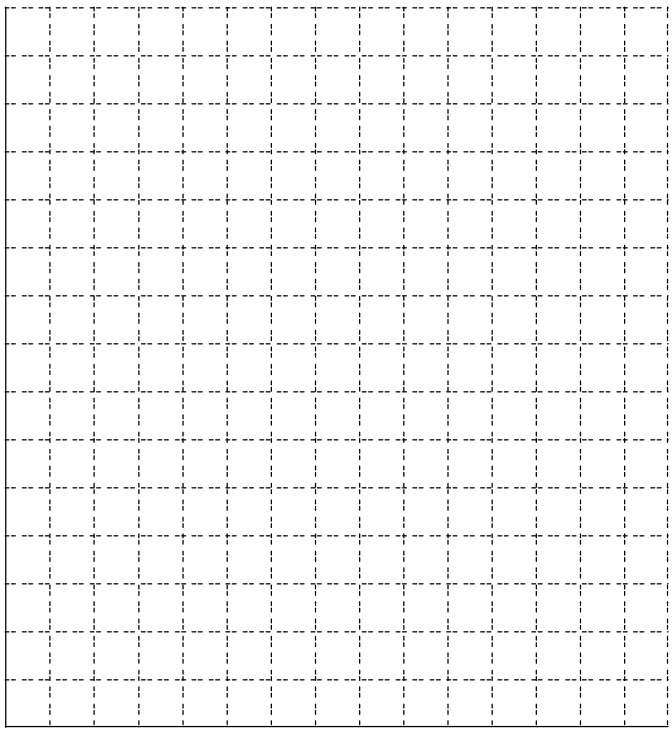
Experiment:

The following data were collected by measuring the amount of bacteria present hourly over a period of 6 hours.

Time (hours)	Bacterial Population
0	5
1	15
2	45
3	135
4	405
5	1215
6	3645

Refer to the Technology Worksheet to perform all calculator features.

1) Create a scatterplot of the data above. (Let the bacterial population be our dependant variable.)
2) Sketch a graph of the scatterplot below. Be sure to include labels and scales.



Student Worksheet #2 – Regression Analysis

3) What function do you think best fits the graph of these data?

4) On your scatterplot above, sketch the curve that best fits your data.

5) Now perform a quadratic regression analysis on the TI-83. Does the regression curve match the curve you drew for #4?

6) Let's look at the data more closely. Complete the table below by finding the first and second differences in the data.

Time (hours)	Bacterial Population	First Difference	Second Difference
0	5	---	---
1	15	10	---
2	45	30	20
3	135		
4	405		
5	1215		
6	3645		

7) If these data were best represented by a quadratic regression, what should we notice in the second difference column?

8) What pattern, if any, appears to be forming in the second difference column?

9) Define the term, **multiplier**, in your own words based on our discussion from #8.

10) How can we use this pattern to try to find the equation of the curve that best fits our data?

11) The standard form of an exponential function is _____.
Now on your TI-83 follow along as we use TI-Interact to find the equation of the curve that best fits these data. Write the equation below.

_____

Student Worksheet #2 – Regression Analysis

*Before moving to the next question press [Y =] and turn off Plot1 by highlighting it and hitting [ENTER] to turn it off. Then press [ZOOM] and choose 6:ZStandard. Now set your values of A and B to 1.

12) Try changing the values of *a* in TI-Interact. What do you notice as your values get larger and smaller?

13) Are there any values that change the graph dramatically?

14) For what value do you first notice a change? How does it change?

15) What does the value of *a* represent in this example? What do you think it generally represents?

16) Now try some values for *b* which are larger than the example. What do you notice as they get larger?

17) Now try some values for *b* which are smaller than the example. What do you notice as they get smaller?

18) The graph of our example represents **exponential growth**. What do you suppose the graph represents when the values of *b* are less than one but greater than zero?

19) Write a mathematical inequality to express the values that describe the two types of exponential functions.

Student Worksheet #3

Below are two sets of data. Data Set 1 is the population of the United States for each specific year recorded in millions of people. Data Set 2 is the annual world crude oil production measured in millions of barrels of oil. Only one of these sets of data can best be modeled by an exponential function.

Data Set 1:

Year	Population (in millions)	Year	Population (in millions)
1815	8.3	1905	83.2
1825	11.0	1915	98.8
1835	14.7	1925	114.2
1845	19.7	1935	127.1
1855	26.7	1945	140.1
1865	35.2	1955	164.0
1875	44.4	1965	190.9
1885	55.9	1975	214.3
1895	68.9		

Data Set 2:

Year	Barrels (in millions)	Year	Barrels (in millions)
1880	30	1940	2150
1890	77	1945	2595
1900	149	1955	5626
1905	215	1960	7674
1915	432	1962	8882
1920	689	1966	12016
1925	1069	1970	16690
1930	1412	1974	20389
1935	1655		

Directions:

1) Using the methods we learned, determine which of the data sets is best modeled by an exponential function. Explain how you arrived at this conclusion.

2) Using the exponential data set, create a scatterplot of the data on your poster paper. Be sure to be neat and accurate. Clearly mark your scales and labels on your scatterplot.

3) Be sure to include a copy of the table of data on your poster paper.

4) Using your table of data, try to estimate the multiplier. Include this estimate and/or work on your poster paper.

5) Find the curve of best fit for the data and write the equation on the poster paper.

6) Sketch the curve of best fit on your scatterplot. Be sure to use a different color.

Student Worksheet #3

7) Be prepared to share with the class how you arrived at your best fit equation. This is not limited to calculator steps. You should be able to justify mathematically as well and be prepared to answer questions.

8) Using what you have learned, make a prediction for what the value was in 1984. Depending on your data set this will be in number of people or barrels. Be able to explain how you arrived at that answer. Show this work on the back of your poster paper.

** BONUS: Find the actual value of the value in 1984. Be sure to include the source site for your answer. How does this answer compare with your predicted answer for #8? Why do you think this is?

Student Worksheet #4 - Homework

Name: _____ Date: _____

I. Identify each function as exponential or non-exponential. BONUS if you can identify what the non-exponential functions are.

1) $f(x) = (77 - x)x$ 2) $g(x) = (2200)^{3.5x}$ 3) $h(x) = 0.5x^2 + 7.5$

II. Determine whether the following functions are exponential growth or decay.

4) $y(t) = 45\left(\frac{1}{4}\right)^t$ 5) $g(x) = 0.25(0.8)^x$ 6) $y(x) = 12(2.5)^x$

7) $f(x) = 722^{-x}$ 8) $h(t) = 45.1\left(\frac{11}{4}\right)^t$ 9) $x(t) = \frac{1}{2}^{-t}$

III. Respond to the following.

10) A population of *E. coli* starts with 55 cells and doubles every hour.

a) What is the multiplier in this situation?

b) Write the exponential function that represents this situation.

c) Predict the population after 5 hours.

11) A certain medication is eliminated from the bloodstream at a rate of about 12% per hour. The initial dose is 40 mg.

a) What is the multiplier in this situation?

b) Write the exponential function that represents this situation.

c) Predict the amount of medication remaining after 3 hours.

Student Worksheet #4 - Homework

12) There are 75 bacteria cells that triple every 30 minutes. What is the bacterial population after 4 hours? Show all work.

13) The compound interest formula is $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$, where P is the principle, r is the annual interest rate, n is the number times interest is compounded per year, and t is the time in years. Find the final amount for each investment showing all work.

a) $750 at 5% interest compounded quarterly for 10 years

b) $1800 at 5.65% interest compounded daily for 6 years

Group Data Worksheet #5

Data:

X	***Y***
0	0.5
1	1.4
2	3.7
3	10.0
4	27.3
5	74.2
6	201.7
7	548.3
8	1490.5

Directions:

1) Create a scatterplot of the data listed above on your poster paper. Be sure to be neat and accurate. Clearly mark your scales and labels on your scatterplot.

2) Be sure to include a copy of the table of data on your poster paper.

3) Using your table of data, try to estimate the multiplier. Include this estimate and/or work on your poster paper.

4) Find the curve of best fit for the data and write the equation on the poster paper.

5) Sketch the curve of best fit on your scatterplot. Be sure to use a different color.

6) Be prepared to share with the class how you arrived at your best fit equation. This is not limited to calculator steps. You should be able to justify mathematically as well and be prepared to answer questions.

7) Using what you have learned, make a prediction for y when x is 10. Be able to explain how you arrived at that answer. Show this work on the back of your poster paper.

Group Data Worksheet #6

Data:

X	***Y***
0	5
1	8.5
2	14.5
3	24.6
4	41.8
5	71.0
6	120.7
7	205.1
8	348.8

Directions:

1) Create a scatterplot of the data listed above on your poster paper. Be sure to be neat and accurate. Clearly mark your scales and labels on your scatterplot.

2) Be sure to include a copy of the table of data on your poster paper.

3) Using your table of data, try to estimate the multiplier. Include this estimate and/or work on your poster paper.

4) Find the curve of best fit for the data and write the equation on the poster paper.

5) Sketch the curve of best fit on your scatterplot. Be sure to use a different color.

6) Be prepared to share with the class how you arrived at your best fit equation. This is not limited to calculator steps. You should be able to justify mathematically as well and be prepared to answer questions.

7) Using what you have learned, make a prediction for y when x is 10. Be able to explain how you arrived at that answer. Show this work on the back of your poster paper.

Group Data Worksheet #7

Data:

X	***Y***
0	0.1
1	0.1
2	0.5
3	2.5
4	12.5
5	62.5
6	312.5
7	1562.5
8	7812.5

Directions:

1) Create a scatterplot of the data listed above on your poster paper. Be sure to be neat and accurate. Clearly mark your scales and labels on your scatterplot.

2) Be sure to include a copy of the table of data on your poster paper.

3) Using your table of data, try to estimate the multiplier. Include this estimate and/or work on your poster paper.

4) Find the curve of best fit for the data and write the equation on the poster paper.

5) Sketch the curve of best fit on your scatterplot. Be sure to use a different color.

6) Be prepared to share with the class how you arrived at your best fit equation. This is not limited to calculator steps. You should be able to justify mathematically as well and be prepared to answer questions.

7) Using what you have learned, make a prediction for y when x is 10. Be able to explain how you arrived at that answer. Show this work on the back of your poster paper.

Group Data Worksheet #8

Data:

X	***Y***
0	0.8
1	2.2
2	5.9
3	16.1
4	43.7
5	118.7
6	322.7
7	877.3
8	2384.8

Directions:

1) Create a scatterplot of the data listed above on your poster paper. Be sure to be neat and accurate. Clearly mark your scales and labels on your scatterplot.

2) Be sure to include a copy of the table of data on your poster paper.

3) Using your table of data, try to estimate the multiplier. Include this estimate and/or work on your poster paper.

4) Find the curve of best fit for the data and write the equation on the poster paper.

5) Sketch the curve of best fit on your scatterplot. Be sure to use a different color.

6) Be prepared to share with the class how you arrived at your best fit equation. This is not limited to calculator steps. You should be able to justify mathematically as well and be prepared to answer questions.

7) Using what you have learned, make a prediction for y when x is 10. Be able to explain how you arrived at that answer. Show this work on the back of your poster paper.

Student Worksheet #1 – Quadratic Regression

Name: **Answer Key** Date:

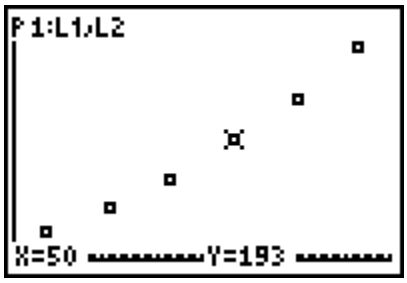
Experiment:

The following data were collected by measuring the stopping distances at varying speeds on dry concrete.

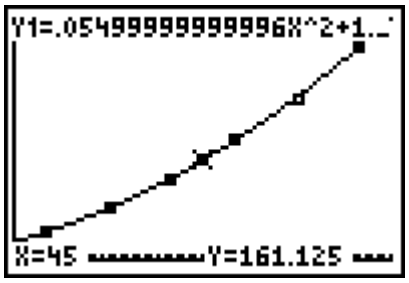
Speed (mph)	Stopping Distance (feet)
20	44
30	83
40	132
50	193
60	264
70	347

Refer to the Technology Worksheet to perform all calculator features.

1) Create a scatterplot of the data above on the TI-83. (Let the stopping distance be our dependant variable.)
2) Sketch a graph of the scatterplot below. Be sure to include labels and scales.



Scatterplot



Scatterplot with regression line

Each scatterplot should be labeled with the following:
***X*-axis: speed (mph) from 20 to 70 possibly by 10's**
***Y*-axis: stopping distance (ft) possibly from 0 to 350 by 50's**

3) What function do you think best fits the graph of these data? **They will probably say quadratic.**

4) Perform a quadratic regression on the data using your TI-83. Sketch the regression line on your graph above and state your regression equation below.

$y = 0.55x^2 + 1.10x + 0.57$ with $R^2 = -.9999$

Student Worksheet #2 – Regression Analysis

Name: **Answer Key** Date:

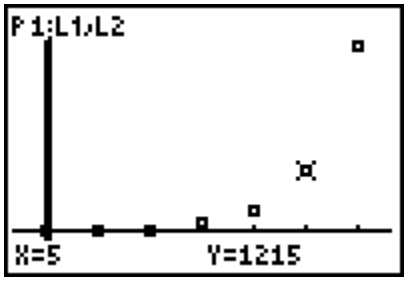
Experiment:

The following data were collected by measuring the amount of bacteria present hourly over a period of 6 hours.

Time (hours)	Bacterial Population
0	5
1	15
2	45
3	135
4	405
5	1215
6	3645

Refer to the Technology Worksheet to perform all calculator features.

1) Create a scatterplot of the data above. (Let the bacterial population be our dependant variable.)
2) Sketch a graph of the scatterplot below. Be sure to include labels and scales.



Graph should be labeled with the following:

***X*-axis: Time (hours) perhaps from 0 to 6 in increments of 1 skipping a space each time on the graph**

***Y*-axis: Bacterial Population perhaps from 0 to 3750 in increments of 250**

Student Worksheet #2 – Regression Analysis

3) What function do you think best fits the graph of these data? **They will probably say quadratic**

4) On your scatterplot above, sketch the curve that best fits your data. **Graphs will vary.**

5) Now perform a quadratic regression analysis on the TI-83. Does the regression curve match the curve you drew for #4? **No, it should not.**

6) Let's look at the data more closely. Complete the table below by finding the first and second differences in the data.

Time (hours)	Bacterial Population	First Difference	Second Difference
0	5	---	---
1	15	10	---
2	45	30	20
3	135	**90**	**60**
4	405	**270**	**180**
5	1215	**810**	**540**
6	3645	**2430**	**1620**

7) If these data were best represented by a quadratic regression, what should we notice in the second difference column? **The value of the second difference should be the same or close to the same value.**

8) What pattern, if any, appears to be forming in the second difference column?
It appears that the second difference is being multiplied by 3 each time to get the next second difference.

9) Define the term, **multiplier**, in your own words based on our discussion from #8.
Definitions may vary. They should mention the fact that it is the number that we are multiplying by each time.

10) How can we use this pattern to try to find the equation of the curve that best fits our data?
Answers will vary.

11) The standard form of an exponential function is ___$y=ab^x$_____.
Now on your TI-83 follow along as we use TI-Interact to find the equation of the curve that best fits these data. Write the equation below.

_____$y = 5 \cdot 3^x$_____

Student Worksheet #2 – Regression Analysis

*Before moving to the next question press [Y =] and turn off Plot1 by highlighting it and hitting [ENTER] to turn it off. Then press [ZOOM] and choose 6:ZStandard. Now set your values of A and B to 1.

12) Try changing the values of *a* in TI-Interact. What do you notice as your values get larger and smaller? **They should notice that the graph moves up and down along the y-axis.**

13) Are there any values that change the graph dramatically? **Any negative numbers.**

14) For what value do you first notice a change? How does it change? **The first value less than zero; the graph reflects over the *x*-axis.**

15) What does the value of *a* represent in this example? What do you think it generally represents? **The original number of bacteria in this example. It represents the initial amount of whatever we are generally examining**

16) Now try some values for *b* which are larger than the example. What do you notice as they get larger? **The graph gets steeper.**

17) Now try some values for *b* which are smaller than the example. What do you notice as they get smaller? **The graph gets less steep. The smaller the number the closer the graph looks like a line. It does not show the graph with negative numbers.**

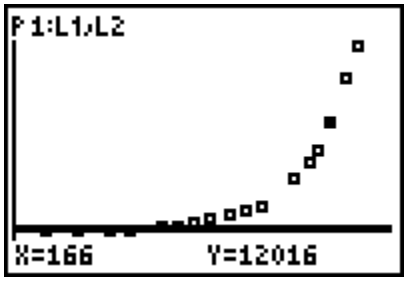
18) The graph of our example represents **exponential growth**. What do you suppose the graph represents when the values of *b* are less than one but greater than zero?
Exponential decay

19) Write a mathematical inequality to express the values that describe the two types of exponential functions.
When $0 < b < 1$ we have exponential growth
When $b > 1$ we have exponential decay

Student Worksheet #3 – Answer Key

1) Of the 2 sets of data, data set 1 is best modeled by a quadratic function and data set 2 is best modeled by an exponential function. Their explanations may vary, but should include something about the multiplier and how there is no constant second difference in the output of data set 2.

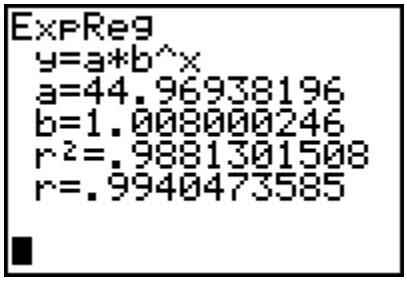
2) The scatterplot of the data should look similar to the graph below. Labels and scales should include: x-axis: Years, perhaps years after 1880 from 0 to 174 by 20's and y-axis: Barrels (in millions), perhaps from 0 to 20500 by 1000's.



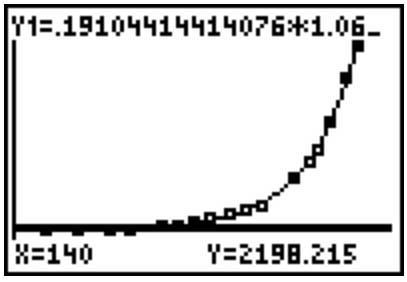
3) Table should be on paper.

4) Multiplier should be close to 1.07. Work should be shown.

5) The equation of the curve of best fit from the TI-83 Plus is shown below.



6) The scatterplot with the curve of best fit from the TI-83 Plus is shown below.



Student Worksheet #3 – Answer Key

7) Have a few students share their answers and compare with the rest of the class.

8) Based on our equation in 1984, (184 years after 1800) the value would be approximately 41,530 millions of barrels of oil. (If the students round their values of *a* and *b* in their equation to hundredths and then substitute, they will arrive at 48,458 millions of barrels of oil.)

BONUS:
The correct value in 1984 was 19,837 millions of barrels of oil. (This value was obtained from the Energy Information Administration.) The reason why this value is so much lower than our predicted value is due to extrapolation. This in essence means that we are trying to predict a value that is outside of the data values that we collected. These values cannot be trusted. Anytime we do a regression analysis we want to make sure we make predictions within our range (or close to our domain values.)

Student Worksheet #4 - Homework

Name: **Answer Key** Date: _____

I. Identify each function as exponential or non-exponential. BONUS if you can identify what the non-exponential functions are.

1) $f(x) = (77 - x)x$

Non-exponential
BONUS: Quadratic

2) $g(x) = (2200)^{3.5x}$

Exponential

3) $h(x) = 0.5x^2 + 7.5$

Non-exponential
BONUS: Quadratic

II. Determine whether the following functions are exponential growth or decay.

4) $y(t) = 45\left(\frac{1}{4}\right)^t$

Decay

5) $g(x) = 0.25(0.8)^x$

Decay

6) $y(x) = 12(2.5)^x$

Growth

7) $f(x) = 722^{-x}$

Decay

8) $h(t) = 45.1\left(\frac{11}{4}\right)^t$

Growth

9) $x(t) = \frac{1}{2}^{-t}$

Growth

III. Respond to the following.

10) A population of *E. coli* starts with 55 cells and doubles every hour.

a) What is the multiplier in this situation? **2**

b) Write the exponential function that represents this situation? $\boldsymbol{Y = 55 \cdot 2^x}$

c) Predict the population after 5 hours. **1760 bacteria**

11) A certain medication is eliminated from the bloodstream at a rate of about 12% per hour. The initial dose is 40 mg.

a) What is the multiplier in this situation? **0.88 = (1 – 0.12)**

b) Write the exponential function that represents this situation? $\boldsymbol{Y = 40 \cdot 0.88^x}$

c) Predict the amount of medication remaining after 3 hours.
Approximately 27.26 mg

Student Worksheet #4 - Homework

12) There are 75 bacteria cells that triple every 30 minutes. What is the bacterial population after 4 hours? Show all work.

$Y = 75 \cdot 3^x$ **4 hours = 8 blocks of 30 minutes**
$Y = 75 \cdot 3^8$
Y = 492075 bacteria

13) The compound interest formula is $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$, where P is the principle, r is the annual interest rate, n is the number times interest is compounded per year, and t is the time in years. Find the final amount for each investment showing all work.

a) $750 at 5% interest compounded quarterly for 10 years

$$\mathbf{A(t) = 750\left(1 + \frac{0.05}{4}\right)^{4(10)}}$$
$$\mathbf{A(t) = \$1232.71}$$

b) $1800 at 5.65% interest compounded daily for 6 years

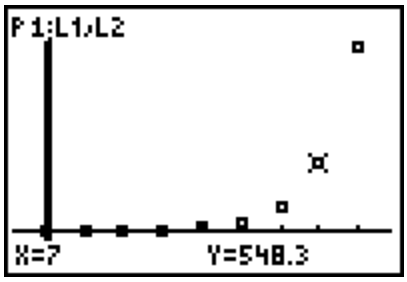
$$\mathbf{A(t) = 1800\left(1 + \frac{0.0565}{365}\right)^{365(6)}}$$
$$\mathbf{A(t) = \$2526.31}$$

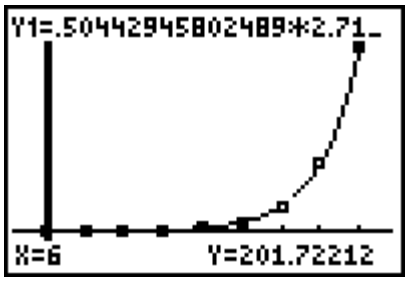
Group Data Worksheet - Answers

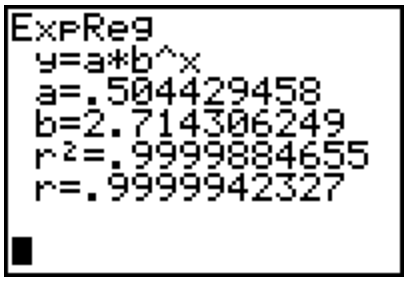
Below is the graphs produced by the TI-83 Plus for the data sets. Obviously there should be labels and scales marked on the graphs of the students. The regression equation that the TI-83 Plus calculated is also included for each data set.

When students are presenting be sure that the students state what their multiplier was and how they found their curve of best fit. They should also explain mathematically how they answered part number 7.

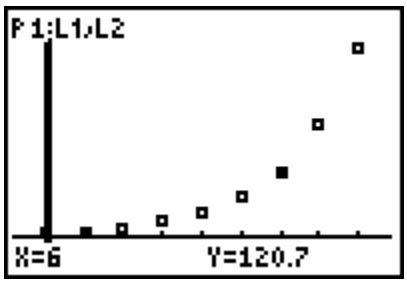
Data Set #5:

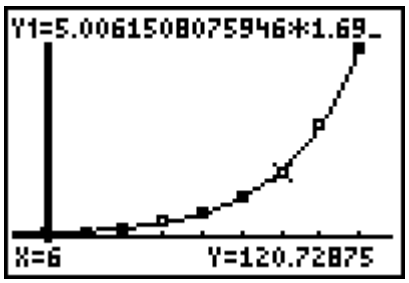


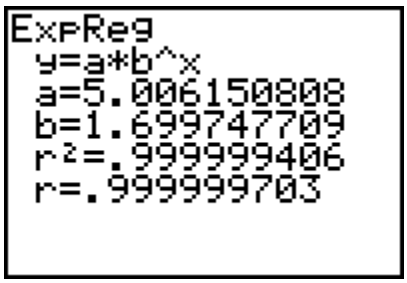




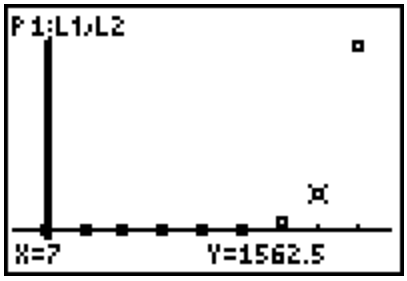
Data Set #6:

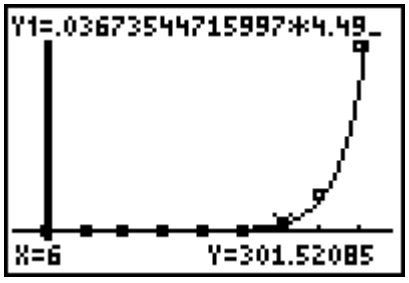


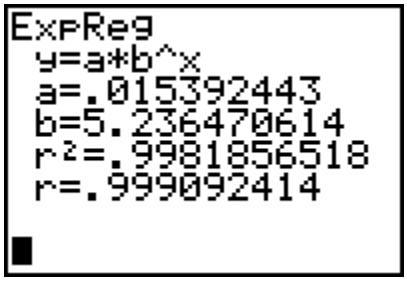




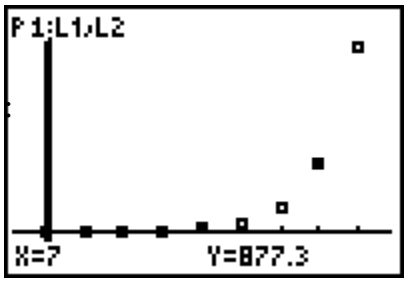
Data Set #7:

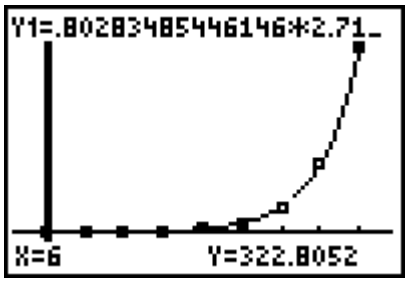


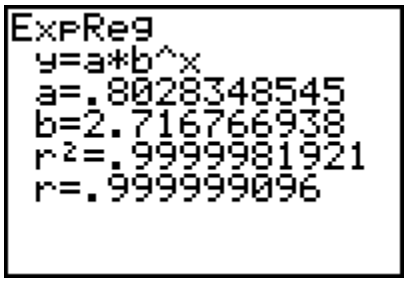




Data Set #8:







Assessment – Exponential Regression

Name: _____ Date: _____

1) Are each of the following exponential? For those that are exponential, identify the multiplier.

a) _____

X	Y
0	-2
3	7
6	34
9	79

b) _____

X	Y
0	5
1	20
2	80
3	320

c) _____

$h(x) = 0.37^x$

d) _____

$h(x) = 0.5x^2 + 7.5$

2) Write an example of an equation for each:

a) Exponential Growth: _____

b) Exponential Decay: _____

3) A population of bacteria starts with 84 cells and doubles every 15 minutes.

a) What is the multiplier in this situation?

b) Write the exponential function that represents this situation?

c) Predict the population after 2 hours.

4) The value of a new car in 1998 is $27,000. It loses 13% of its value each year. Write and evaluate an expression to estimate the car's value in 2006.

Assessment – Exponential Regression

5) **ECR:** The following data represents the number of new software patents that are granted in the US each year since 1971.

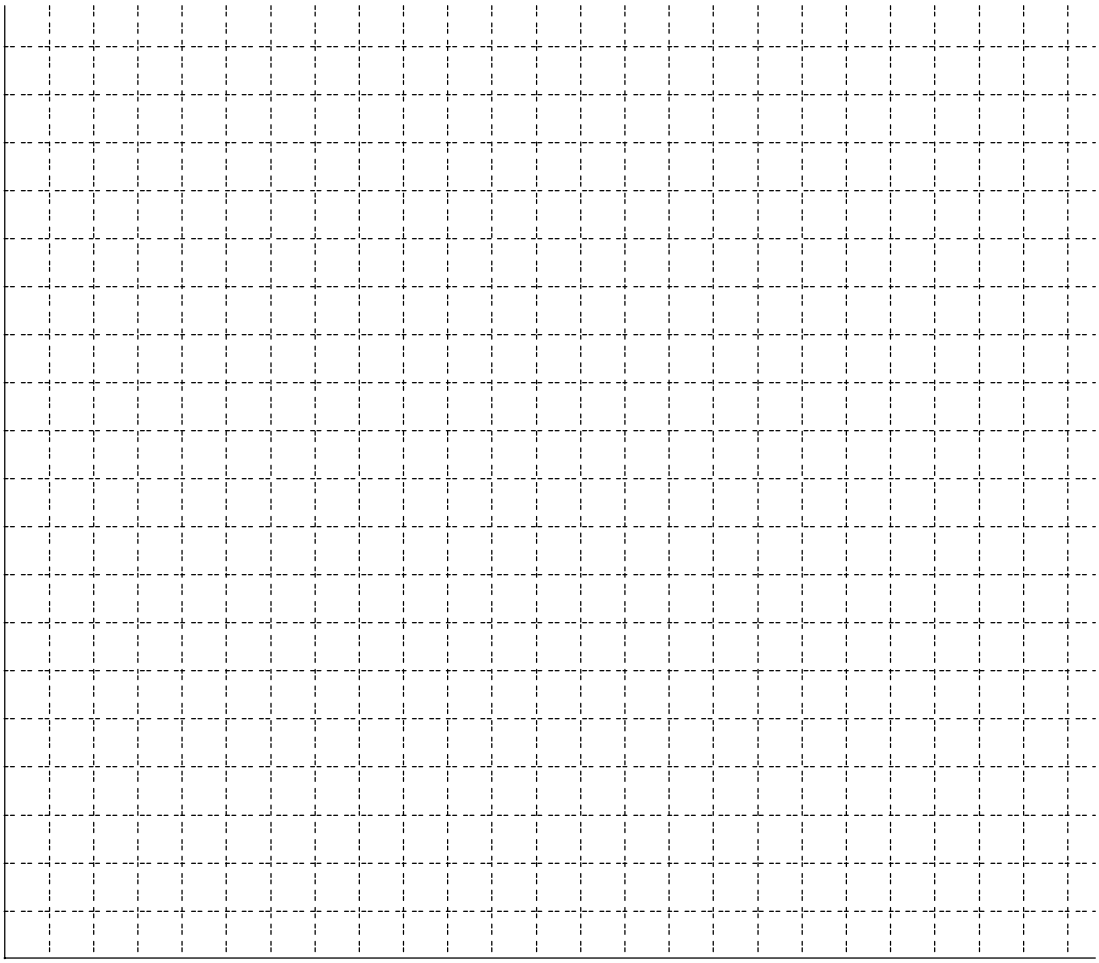
Year (t)	Number of Patents (P)	Year (t)	Number of Patents (P)
1971	896	1983	1517
1972	906	1984	1857
1973	871	1985	1978
1974	838	1986	2202
1975	817	1987	2766
1976	1113	1988	2708
1977	1320	1989	3980
1978	1272	1990	3606
1979	986	1991	3817
1980	1239	1992	4073
1981	1257	1993	4862
1982	1446		

- Using the data in the table above, draw a scatterplot on the grid provided (attached).

- Write an equation that represents the number of patents granted in the US (P) for each year (t). Explain how your determined your equation using mathematical words, symbols or both in your explanation.

- Predict the number of patents granted in the US in the year 1995. Use mathematics to justify your answer.

Assessment – Exponential Regression



HSA Mathematics Rubric
Extended Constructed Response Items

	Application	Representation	Explanation	Justification	Analysis
4	Application of a reasonable strategy that leads to a correct solution in the context of the problem.	Representations are correct.	Explanation is logically sound, clearly presented, fully developed, supports the solution, and does not contain significant mathematical errors.	Justification that is logically sound, clearly presented, fully developed, supports the solution, and does not contain significant mathematical errors.	The response demonstrates a complete understanding and analysis of the problem.
3	Application of a reasonable strategy that may or may not lead to a correct solution.	Representations are essentially correct.	Explanation is generally well developed, feasible, and supports the solution.	Justification is generally well developed, feasible.	The response demonstrates a complete understanding and analysis of the problem.
2	Incomplete application of a reasonable strategy that may or may not lead to a correct solution.	Representations are fundamentally correct.	The explanation supports the solution and is plausible, although it may not be well developed or complete.	Justification supports the solution and is plausible, although it may not be well developed or complete.	The response demonstrates a conceptual understanding and analysis of the problem.
1	Little or no application of a reasonable strategy. May or may not have the correct answer.	Representations are incomplete or missing.	Reveals serious flaws in reasoning. May be incomplete or missing.	Reveals serious flaws in reasoning. May be incomplete or missing.	The response demonstrates a minimal understanding and analysis of the problem.
0	The response is completely incorrect or irrelevant. There may be no response, or the response may state, “I don't know.”				

Explanation refers to the student using the language of mathematics to communicate how the student arrived at the solution.
Justification refers to the student using mathematical principles to support the reasoning used to solve the problem or to demonstrate that the solution is correct. This could include the appropriate definitions, postulates and theorems.
Essentially correct representations may contain a few minor errors such as missing labels, reversed axes, or scales that are not uniform.
Fundamentally correct representations may contain several minor errors such as missing labels, reversed axes, or scales that are not uniform.

Assessment – Exponential Regression

Name: **Answer Key** Date:

1) Are each of the following exponential? For those that are exponential, identify the multiplier.

a) **no**

X	Y
0	-2
3	7
6	34
9	79

b) **yes**
4

X	Y
0	5
1	20
2	80
3	320

c) **yes**
0.37
$h(x) = 0.37^x$

d) **no**
$h(x) = 0.5x^2 + 7.5$

2) Write an example of an equation for each:

a) Exponential Growth: _ **answers may vary, must have a multiplier greater than one**

b) Exponential Decay: _ **answers may vary, must have a multiplier less than one but greater than zero**

3) A population of bacteria starts with 84 cells and doubles every 15 minutes.

a) What is the multiplier in this situation? **2**

b) Write the exponential function that represents this situation? $\mathbf{Y} = 84 \bullet 2^x$
where *x* represent the number of 15 minute intervals passed

c) Predict the population after 2 hours. **21504 bacteria**

4) The value of a new car in 1998 is $27,000. It loses 13% of its value each year. Write and evaluate an expression to estimate the car's value in 2006.

***Y* = 27000·0.87^8 = $8861.72 The multiplier is 1-0.13=0.87. Since 2006 is 8 years after 1998, we use 8 as the years past.**

Assessment – Exponential Regression

5) **ECR:** The following data represents the number of new software patents that are granted in the US each year since 1971.

Year (t)	Number of Patents (P)	Year (t)	Number of Patents (P)
1971	896	1983	1517
1972	906	1984	1857
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1974	838	1986	2202
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1978	1272	1990	3606
1979	986	1991	3817
1980	1239	1992	4073
1981	1257	1993	4862
1982	1446		

- Using the data in the table above, draw a scatterplot on the grid provided (attached).

- Write an equation that represents the number of patents granted in the US (P) for each year (t). Explain how your determined your equation using mathematical words, symbols or both in your explanation.

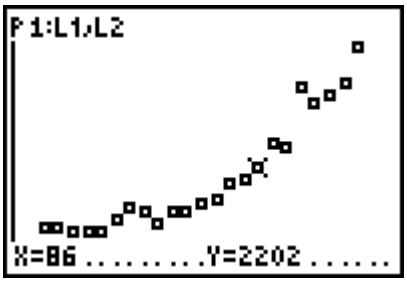
- Predict the number of patents granted in the US in the year 1995. Use mathematics to justify your answer.

  Use attached rubric to score this question. Correct answers follow below on the next page.

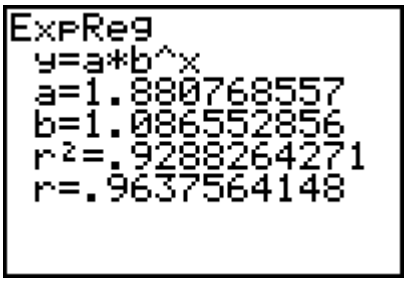
Assessment – Exponential Regression

The graph of the scatter plot should look like this:

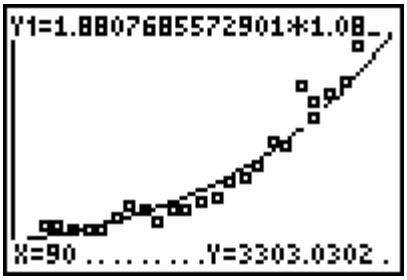
(Labels on the *x*-axis should be years after 1900 with data starting at 71 and going through 93. The *y*-axis should be labeled number of patents with the smallest value graphed at 817 and the largest 4862. Scales may vary)



The equation of the exponential regression line is this:



This is the scatterplot with the regression line on top:



With this regression equation rounded to the nearest hundredth, the estimate of the number of patents granted in the US in the year 1995 would be 6756 patents.

Work: $P(t) \approx 1.88 \bullet 1.09^{t}$

$P(t) \approx 1.88 \bullet 1.09^{95}$

$P(t) \approx 6756$

However, when you use the regression equation from the TI-83 you get 5002 patents.